Abstract

Game-theory is a field of mathematics that can be used to study the formation of equilibrium prices in economic markets. Although the validity of market equilibrium as an empirical concept is much debated, equilibrium theory can still provide valuable insights for pricing strategy as it provides guidelines for rational decision making in highly competitive markets. In this study we develop a game-theoretic approach to optimal pricing when markets are characterized by strong competition between rational decision makers having complete information. A model is developed that describes the process of mutual competition between players (e.g. hotels) through their market share functions where each of these functions is described as a multinomial logit model with an additive utility function. Using an additive utility function allows for the players’ prices to be different in the same equilibrium which reflects the notion that high-end hotels with many attractive facilities can justify higher prices but can still be considered to be in an equilibrium. Choice-based conjoint analysis is used to provide the attributes weights (“part-worth’s”) for the utility function. The approach is illustrated by an example.

Keywords: Revenue Management, Pricing, Conjoint Analysis, Equilibrium, Discrete Choice Model, Game Theory

Introduction

The concept of market equilibrium is a central concept in price theory (Bridel, 2001). It represents the notion that competitor prices in a given market necessarily converge to a steady state where the resulting prices are, in some sense, “optimal”. This optimality is generally defined as being such that firms have no economic incentive to change prices from equilibrium levels as that would decrease their profits. Through time, however, the concept of market equilibrium has also received academic criticism. As early as Edgeworth (1881), doubts were casted about the stability and convergence of equilibria. Von Hayek (1937, pp. 43-44) even stated ‘We shall not get further here […] the only justification for this is the supposed existence of a tendency toward equilibrium […] ceases to be an exercise in pure logic’. In other words, whilst theoretically appealing, the mode of motion and state of an equilibrium are actually quite hard to verify empirically. The stock market for example, arguably the most archetypical of all markets, can hardly be considered to be in a state of equilibrium. Or, as the application of game theory has showed, whilst is it possible to theorize numerous different market equilibria, they can be all be mirrored by an equally rich array of behavioural patterns actually observed in markets. In fact, by explicitly recognizing the wide range of behaviour predicted by theory and actually observed, many economists have concluded that the equilibrium problem (e.g. plausibility, existence, uniqueness, stability) is
indeterminate (Vives, 1999). Therefore, a general observation across different types of markets has led many to believe that the concept of market equilibrium has only limited validity as a real-world phenomenon but instead should be considered more as a normative theory (Fog, 1994). That is, it should be considered as a general theory for how markets should behave in order to be efficient. An interesting question then is whether equilibrium theory, viewed from the perspective of game theory, has something to offer to decision-makers on a practical micro-level, for example to revenue managers in the hospitality sector.

This paper explores the potential benefits of applying game theory together with conjoint analysis to the practical pricing decision of revenue managers in the hospitality industry. First of all, a basic game theoretical equilibrium model is introduced. Then a mathematical model is developed to describe the process of strategic pricing between hotels in a closed market. A key element in this model is that each revenue manager’s profit is driven by his market share which, in turn, is defined as a function of both the manager’s own price setting as well as his competitor’s. Furthermore, it is assumed that hotels also compete on other attributes than price, such as quality, location, service, etc. It is also assumed that hotels with a good performance on these product and service attributes (from a guest perspective) create a certain degree of customer value which justifies a higher price level (compared to competing hotels with a lower performance). In this way, market prices might be markedly different from each other and might still be considered to be in an equilibrium state. To incorporate such complex attribute trade-off behaviour by hotel guests, the market share in the model is further operationalized through a discrete choice model, the parameters of which are to be measured by use of choice-based conjoint analysis.

Integrating conjoint analysis with game-theoretic equilibrium principles has received some attention in the literature. This paper is an extension of the work by Choi and DeSarbo (1993) who develop a general framework in the context of competing producers of car-tires. The current paper is different in a number of ways. Firstly, it utilizes a choice-based conjoint approach instead of a traditional full profile conjoint approach which not only brings the model up to date with modern standards in conjoint analysis practice but more importantly enables the use of a so-called “none-option” in the conjoint model in order to model any (exogenous) aggregate competitive effects not covered by the endogenous players. The paper will show that this is a crucial step towards the empirical validation of the model. Secondly, the current paper focuses on price as the only (continuous) variable that can be influenced by the decision makers and treats the other (discrete) attributes as fixed. This is in contrast to the approach by Choi and DeSarbo (1994) where all the attributes (continuous and discrete) are to be optimized. The simplification is acceptable as price is the main variable of interest to the revenue manager. Furthermore, by focusing on price as the only continuous independent variable the paper can simplify the estimation procedure and also eliminate the somewhat troubling possibility of the existence of multiple equilibria. The current paper aims at providing the theoretical derivation of the model as a precursor to empirically verifying its validity.

A Basic Game Theoretical Equilibrium Model

Game theory, an active field of mathematics studying the strategic interaction of rational decision makers, provides a general framework for describing managerial decision making in the face of market equilibria. Central concepts in game theory are Players (strategic decision makers), Payoffs (the benefits the decision makers strive to acquire) and Strategies (the planned sequences of activities to acquire these payoffs). Variations on these three variables yield a huge variation of game typologies, suited to model all different kinds of real-world phenomena. Arguably the best known example of these is the Prisoner’s Dilemma where two Players (prisoners A and B accused of jointly committing a crime and being interrogated by the police) strive to obtain either one of two conflicting Payoffs (High Sentence vs. Low Sentence) by using either a cooperative (stay silent) or non-cooperative (confess), one-round-off-play Strategy. This is described in figure 1.
As for many strategic games the Prisoner’s Dilemma can explain a great deal about what actually constitutes a “good” decision in such a situation, namely that in this case it is actually wisest not to cooperate. As a Player knows that the opponent is a rational decision maker (and thus that this opponent will implement a strategy that is optimal for himself), the Player’s own optimal strategy would be not to cooperate. After all, no matter what the opponent does, the Player’s own optimal strategy will always be not to cooperate. The inevitable outcome of this (at least theoretically) is that both Players confess and indeed this is the very mechanic that police interrogators use in practice to get criminals behind bars. This theoretically “inevitable” outcome constitutes a so-called Nash equilibrium (Nash, 1950).

**Towards a Managerial Framework**

How does all this relate to the daily routine of a revenue manager in the hospitality business? In a way, setting prices in a competitive market can be viewed as a strategic game where the players are the revenue managers of the hotels in a given market, the payoffs are the revenues resulting from a certain price setting and the players’ strategies are to set their room prices in such a way as to optimize their own revenues. The most important difference with a Prisoner’s Dilemma game is that the payoffs would not be defined as binary outcomes (cooperate vs. not cooperate) but instead as continuous functions (revenue as a function of price). For example if total market demand is assumed to be fixed in the short run, then a very general payoff function for the revenue manager of a hotel i in a market with only two hotels (hotel i and j) might be postulated as:

\[ R_i = DM_i(P_i, P_j)P_i \]  

(1)

where:

- \( R_i \) = revenue for hotel i;
- \( D \) = total market demand;
- \( P_i \) = room price charged by hotel i;
- \( P_j \) = room price charged by hotel j;
- \( M_i(P_i, P_j) \) = market share for hotel i as a function of room prices charged by hotel i and j;

This payoff function states that the total payoff a revenue manager is going to “yield” from a certain pricing strategy (fully defined by the manager of hotel i setting his price at a level \( P_i \)) is equal to a fixed total market demand \( D \) times the market share he is going to obtain at price \( P_i \) times price \( P_j \). Because his market share depends on both his own price and the price set by his competitor, market share is defined as a function of both \( P_i \) and \( P_j \).

The next step is to determine the Nash equilibrium-price if the two revenue managers would act rationally upon their payoff functions. The optimum occurs at the value of \( P_i \) where the first derivative of (1) with respect to \( P_i \) is equal to zero. That is:

\[ \frac{dR_i}{dP_i} = 0 \]  

(2)

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1. This is true irrespective of the fact that both players would have been better off if they both would have stayed quiet.
2. The subsequent derivation is true for any number of players without loss of generality. Note that we use “hotel i” and “the manager of hotel i” as interchangeable identifiers.
or, using the product rule of differentiation:

\[ P_i \frac{dM_i(P_i,P_j)}{dP_i} + M_i(P_i,P_j) = 0 \]  \hspace{1cm} (3)

Solving equation (3) for \( P_i \) gives the optimal (revenue-maximizing) price for hotel i. It should be noted however, that (3) only has a unique solution if the price for hotel j, \( P_j \), is known. If \( P_j \) is not known then equation (3) has two unknowns and consequently is not identified. This makes sense: a revenue manager will not be able to make an informed pricing decision if he doesn’t know what price the opponents are charging. Although a revenue manager may not know the opponent’s exact price, what can be assumed is that a rational revenue manager will also be maximizing revenues via a similar, but opposite, revenue function, that is:

\[ P_j \frac{dM_j(P_i,P_j)}{dP_j} + M_j(P_i,P_j) = 0 \]  \hspace{1cm} (4)

Both (3) and (4) define the optimal price of one player given the optimal price of the other, and this is exactly what constitutes a Nash equilibrium. Because (3) and (4) together form a system of two equations with two unknowns, this system is identifiable. Solving (3) and (4) simultaneously will yield the vector \( \{P_i^*,P_j^*\} \) which defines the resulting optimal prices in a unique Nash equilibrium.

The practical implication of \( \{P_i^*,P_j^*\} \) is that at these prices, neither player has an incentive to change his price because this will always result in a decrease of his revenue. Although this mathematical fact derives directly from the formulations, some interesting observations can be made about the empirical side of this. That is, the existence of \( \{P_i^*,P_j^*\} \) says nothing about how the players should arrive at them. Formally, the Nash equilibrium refers to a single-play game in which the players set their prices once and only once. In reality however, players dynamically iterate prices back and forth and may converge to this equilibrium only over time. In fact, they might just as well arrive at an altogether different equilibrium or maybe even to no equilibrium at all (in which case prices remain in constant flux as can be observed in modern stock markets for example). It cannot be overstated that game theory in itself has nothing to say about whether equilibria should or shouldn’t be observed in reality. It only predicts what outcomes would be rational. Whether Nash equilibria are actually observed in the real-world is ultimately dependent on whether players act rationally and whether they have complete information about the other players’ strategies. In the real world players have to deal with limited information, time-pressured decision making, cognitive limitations of the mind, and inter-organizational politics. It was for this reason that Simon (1955) proposed the notion of bounded rationality as an alternative basis for mathematical modeling in economic decision-making and that Kahneman and Tversky (1979) proposed the use of heuristics as the basis for human decision making.

The Operationalization of Market Share

Equation (1) conveniently ignored the dynamics behind the formation of market share. However, in order to be able to solve the system of equations in (3) and (4) for their respective prices \( P_i^* \) and \( P_j^* \) it will be necessary to account for it, as it still constitutes an unknown function.

A convenient and very general way of modeling market share is by the use of the Multinomial Logit model (MNL). In applied choice analysis the MNL is formulated as a model of individual decision making as:

\[ Pr(Y_q = i) = \frac{\exp(U_{qi})}{\exp(U_{qi}) + \exp(U_{qj})} \]  \hspace{1cm} (5)
where:

\[ Pr(Y_q = i) \] = the probability that a decision maker \(^3\) \(q\) will choose a concept \(i\) from a set \(\{i, j\}\),
\[ U_{qi} \] = the utility that decision maker \(q\) associates with alternative \(i\),
\[ U_{qj} \] = the utility that decision maker \(q\) associates with alternative \(j\).

Formula (5) can be derived under quite general conditions from the assumption that a decision maker \(q\) will choose a concept \(i\) from a set \(\{i, j\}\) if and only if:

\[ U_{qi} + \varepsilon_{qi} > U_{qj} + \varepsilon_{qj} \] \hspace{1cm} (6)

Here, “utility” has the conventional economic interpretation of the amount of well-being that a consumer \(q\) expects to derive from the consumption of a good \(i\). The MNL model in (5) is derived from (6) under the basic axioms of rational choice theory and the additional assumption that the errors \(\{\varepsilon_{qi}, \varepsilon_{qj}\}\) are distributed as i.i.d. Extreme Value Type 1 (see McFadden (1974) and Louviere, Hensher and Swait (2001) for a full derivation of equation (5)).

Under homogenous market conditions the market share of a product can be viewed as the probability that a “representative” consumer chooses that product from the set of available products. If this assumption is reasonable, then \(Pr(Y_q = i)\) might be replaced by \(M_i\) and consequently (5) becomes:

\[ M_i = \frac{\text{Exp}(U_{qi})}{\text{Exp}(U_{qi}) + \text{Exp}(U_{qj})} \] \hspace{1cm} (7)

At this point the utility in (7) is no longer defined as the expected well-being for an individual consumer, but instead as that for an “average” consumer in the market. In practice, revenue managers can use statistical techniques such as choice-based conjoint analysis to estimate \(U_{qi}\) and \(U_{qj}\) from a set of representative behavioural data. This will be discussed in the next section. Note that the dynamics of (7) are intuitively plausible: market share for a product increases when it becomes more desirable or when a competitors’ product becomes less desirable, and vice versa. Furthermore, market share is constrained to lie within the \(<0, 1>\) interval. It can thus be argued that (7) provides a well-behaved and plausible model of customer choice within a competitive context.

Up until now the behavioural model is very similar to that in Choi and Desarbo (1993). However, an important advancement can be made by adding a so-called “no-choice option” to the model. As can be seen from the formulation in (7) the model is fairly restricted in the sense that the decision maker is forced to make a choice between the available options without the possibility to choose no product at all. However, it is quite reasonable to expect that a consumer might want to defer his choice if neither of the options available to him is good enough. In line with (6) it could be assumed that a certain option is only good enough for choosing if its utility exceeds a certain utility threshold, that is if and only if:

\[ U_{qi} + \varepsilon_{qi} > N_q + \varepsilon_{qn} \] \hspace{1cm} (8)

where \(N_q\) is the threshold utility for choice by decision maker \(q\) in a given market (also called the “no-choice utility”). The no-choice option reflects the decision to never buy a product at all or, alternatively, the decision to postpone a choice to some future point in time when product availability might change and the prospect of obtaining a desirable product might improve. In the latter case the no-choice option reflects an aggregate measure of attractiveness of the competitive offerings that exist outside the current choice occasion.

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\(^3\) Note the distinction between the “decision maker” in the previous sections (which is the player in the game; i.e. the hotel manager) and the decision maker in the context of MNL, which is the hotel guest.
The inclusion of a no-choice option in a game-theoretic framework of discrete choice models is not only relevant from a behavioural perspective. It is also a crucial step towards enabling the empirical validation of such a model. If the competition between two hotels is modeled without a no-choice alternative then every guest would be forced to make a choice between the two hotels no matter how high the prices actually beget. The optimal prices \( P_i^*, P_j^* \) would be determined by solving (3) and (4), but without a constraint on total market demand this would lead to both hotels excessively raising their prices as they would only have to worry that guests change to the other player and not that they will leave the market altogether. Whilst this may be a reasonable assumption in a few oligopolistic commodity markets (e.g. gasoline) it is unrealistic for hospitality markets with their immensely fragmented structure where it is unlikely that the full set of options available to the decision maker can be included in any choice model, no matter how well defined. The amount of monopoly power in hospitality markets, however, is much greater than a superficial analysis of market structure might imply. This is so in part because of very strong product differentiation and in some fields cartel-like restrictions on entry (Scherer and Ross, 1990). Nevertheless, the inclusion of a no-choice utility is necessary in order to realistically model the price formation process in hospitality markets. By their very nature, hospitality markets have an almost unlimited number of choice alternatives available (e.g. venues, locations, substitutes, postponement) which cannot be included as individual players in a game.7

Using conjoint analysis to measure the utility contribution of hotel attributes

Having defined a general structure for the competition between the players’ propositions and with the no-choice alternative in place, the product-level utilities are now to be specified. This can be done using an additive utility model (Fishbein, 1967) as follows:

\[
U_i = \sum_{k=1}^{K} (B_k X_{ki})
\]

That is, a total of \( K \) attributes are postulated by the revenue manager as being relevant to the formation of product utility within a certain product market. A particular offering \( i \) is then described by an attribute vector \( \{X_{ki}\} \), each being weighted by a vector of parameters \( \{B_k\} \). This vector of taste parameters should be considered a market average of the taste parameters of the individual customers therein. For all practical purposes \( \{B_k\} \) can be estimated using such market research techniques as (choice-based) conjoint analysis (for stated choice data) or discrete choice analysis (for revealed choice data).

In neoclassical price theory, \( K \) is equal to 1. That is, price is the only relevant attribute. And, if no other attributes would be defined, this would be the endpoint of the model derivation as functions (3) and (4) could now be derived with only the prices of the players as the remaining unknowns (as will be shown later on the level of market demand \( D \) is irrelevant to the location of the equilibrium). However, it will be much more interesting if other terms are added to the utility function. With regard to the hospitality industry for example, one could easily include quality aspects such as room square footage, friendliness of staff, distance to the airport, quality of food in the hotel restaurant, classiness of the interior of the hotel, tidiness and cleanliness of the hotel, and so forth, as additional attributes. The possibility of adding quality attributes to the hotel utility function is not only interesting from the point of view of model realism. More importantly, it offers a fascinating perspective on the formation of equilibrium prices. If (9) would contain only price as an attribute then in the Nash equilibrium the prices of the competing hotels would be exactly equal \( P_i = P_j \). In other words, there is nothing that sets one hotel apart from the other if price would be their only distinguishing characteristic. After all, (1), (7) and (8) are indistinguishable for the two players except for the indexes. On the other hand, using additional attributes in the utility function would allow for differentiated price competition between hotels where higher prices might be justified by higher quality or other desirable characteristics.

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7 Because of their choice of conjoint method (i.e. traditional full profile rating/ranking) Choi and deSarbo (1994) were unable (in a methodological sense) to estimate a no-choice utility. For this, choice-based conjoint analysis or any other discrete choice analysis procedure is needed.

5 Note that the use of the MNL model for the market share formation precludes the inclusion of a constant in the utility function. The MNL model is invariant under the addition of a single constant to every player’s utility function.
Conjoint analysis offers a very natural way of measuring the weight that consumers attach to different attributes associated with a certain product or service and it can be used to fill in the $\{B_k\}$. This then completes the model formulation and leaves the model to be estimated, which will be illustrated in the next section.

**Illustration**

In order to illustrate the methodology imagine a tropical island with exactly two 3-star hotels (of comparable quality but one has a swimming pool while the other has not) and a backpacker’s hostel on it. The island constitutes a “closed” market in the sense that any visitor that plans to visit the island has to choose one and only one of the three available options. Of course, if none of the three options are good enough (in terms of either quality or price) the guest might decide not to visit the island at all. Our objective would be to determine the optimal equilibrium prices for both hotels and the backpacker’s hostel given their mutual competition.

The first step would be to conduct a choice-based conjoint experiment on the island. A representative sample of island visitors would be interviewed (for example in the lobbies of the three venues) and given a number of conjoint choice tasks to complete. The product attributes that determine customer choice in this fairly simple example could simply be Type of venue (3-star hotel vs. hostel), Swimming pool (yes vs. no) and Price (50, 75, and 100 euros per night). Additionally, a none-option would be presented to allow the respondent to opt out of visiting the island if none of the alternatives would be acceptable to him. An example of a choice task that is consistent with this setup is shown in figure 2.

Figure 2: example choice task for the visitors of the tropical island.

![Example Choice Task](image)

After completion of the fieldwork we would proceed by fitting a basic MNL model with simple additive utility (as described by formulas 5 through 9) to the experimental data. Assume that table 3 reflects the estimated coefficients (“part-worth’s”) of such a model.

Table 3: estimated coefficients for MNL model with additive utility function.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Level</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of venue</td>
<td>3-star hotel</td>
<td>3.50</td>
</tr>
<tr>
<td></td>
<td>hostel</td>
<td>0.00</td>
</tr>
<tr>
<td>Swimming pool</td>
<td>yes</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>no</td>
<td>0.00</td>
</tr>
<tr>
<td>Price</td>
<td>(multiplier)</td>
<td>-0.02</td>
</tr>
<tr>
<td>None</td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

In this model, Type of venue and Swimming pool are nominal attributes coded as dummy variables with the least attractive level coded as the base value (0). In contrast, Price is coded as a scale variable, that is: the partial utility of a price at level $X$ enters the utility function in (9) through a linear function as $-0.02X$ (off course the
model could easily be adapted to handle non-linear relationships and interactions as well). The coefficients in table 3 are reasonable if we assume that most people would prefer a hotel over a hostel, a swimming pool over no swimming pool and lower prices over higher prices. The value of 1 for “None” reflects the tendency of people not to choose any of the hotels available when they are unacceptable to them.

The next step would be to construct the player’s market share functions as described by formula 7. If we index the players by \( i = 1 \ldots 3 \) where \( i = 1 \) is the hotel with swimming pool, \( i = 2 \) is the hotel without swimming pool and \( i = 3 \) is the hostel (also without swimming pool), then their respective market share functions would be:

\[
\begin{align*}
M_1(P_1,P_2,P_3) &= \exp(3.5+0.5-0.02P_1)/(\exp(3.5+0.5-0.02P_1)+\exp(3.5+0-0.02P_2)+\exp(0+0-0.02P_3)+\exp(1.0)) \\
M_2(P_1,P_2,P_3) &= \exp(3.5+0-0.02P_2)/(\exp(3.5+0.5-0.02P_1)+\exp(3.5+0-0.02P_2)+\exp(0+0-0.02P_3)+\exp(1.0)) \\
M_3(P_1,P_2,P_3) &= \exp(0+0-0.02P_3)/(\exp(3.5+0.5-0.02P_1)+\exp(3.5+0-0.02P_2)+\exp(0+0-0.02P_3)+\exp(1.0))
\end{align*}
\]  

In order to specify (3) we also need the first derivatives of these functions with respect to \( P_i \) which are given by:

\[
\begin{align*}
P_1 \times M_1(P_1,P_2,P_3) \times (1-M_1(P_1,P_2,P_3)), \\
P_2 \times M_2(P_1,P_2,P_3) \times (1-M_2(P_1,P_2,P_3)), \\
P_3 \times M_3(P_1,P_2,P_3) \times (1-M_3(P_1,P_2,P_3)).
\end{align*}
\]

Together, equations (10) through (15) provide all the required inputs to construct the first derivatives of the payoff functions with respect to \( P_1 \) as described in (3) and subsequently set them to zero, that is:

\[
\begin{align*}
P_1 \times (P_1 \times M_1(P_1,P_2,P_3) \times (1-M_1(P_1,P_2,P_3))) + M_1(P_1,P_2,P_3) &= 0, \\
P_2 \times (P_2 \times M_2(P_1,P_2,P_3) \times (1-M_2(P_1,P_2,P_3))) + M_2(P_1,P_2,P_3) &= 0, \\
P_3 \times (P_3 \times M_3(P_1,P_2,P_3) \times (1-M_3(P_1,P_2,P_3))) + M_3(P_1,P_2,P_3) &= 0,
\end{align*}
\]

with the \( \{M_i(P_1,P_2,P_3)\} \) as previously defined in (10) through (12). In order to solve this system of equations, that is to find the values \( \{P_1^*,P_2^*,P_3^*\} \) where all the functions are equal to zero, a modified Newton-Raphson procedure is implemented. The Newton-Raphson procedure is an iterative algorithm to estimate the root of a function (i.e. the value \( X \) at which \( f(X) \) evaluates to zero) by using the updating formula:

\[
X_{t+1} = X_t - f(X_t)/ f'(X_t),
\]

where \( f(X_t) \) is the function of which the root is to be found, in this case being (16) through (18), and \( f'(X_t) \) is its first derivative with respect to \( X_t \). Because the first derivatives of (16) through (18) are quite laborious to calculate analytically it is suggested to approximate them by calculating the gradient at \( X_t \) by forward finite differencing. The final estimation algorithm thus becomes:

**Step 1:** Set initial values at \( P_1=10, P_2=10, P_3=10 \). Set step size for finite differencing at \( k=0.01 \).

**Step 2:** Calculate the left hand sides of (16) through (18) and store these values as vector \( V_1 \).

**Step 3:** Increase \( \{P_1,P_2,P_3\} \) by step size \( k \).

**Step 4:** Calculate the left hand sides of (16) through (18) and store these values as vector \( V_2 \).

**Step 5:** Approximate the first derivative of (16) through (18) as: \( (V_2-V_1)/k \) and store as vector \( V_3 \).

**Step 6:** Update \( \{P_1,P_2,P_3\} \) based on the Newton-Raphson formula in (19) using \( V_1 \) and \( V_3 \).

**Step 7:** Repeat step 2 through 6 until \( \{P_1,P_2,P_3\} \) converge to their equilibrium prices at \( \{93,79,51\} \).

The algorithm described above typically converges within about 30 iterations and is very easy to implement in a standard spreadsheet program. Although the algorithm is fairly stable under varying starting conditions, it should always be repeated from multiple starting points in order to confirm that the right results have been obtained.

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\(^6\) Note that by inclusion of a none-option the \( \{M_i\} \) no longer refer to “market shares” in a strict sense as market shares, by definition, sum to one over the players only. Instead, the \( \{M_i\} \) sum to one over the players plus the no-choice option. Nonetheless, for the sake of clarity the \( \{M_i\} \) shall still be referred to as market shares.
The equilibrium prices \{93, 79, 51\} reflect the optimal prices for the two hotels and the hostel because a unilateral price change by any of the players will always (at least mathematically) lead to a lower revenue for this player. Therefore neither player has an incentive to change his price from this optimal level (Nash equilibrium).

Note that the prices \{93, 79, 51\} constitute an equilibrium irrespective of the fact that they are different from one another. Hotel i=1, with its 3-star facilities and its swimming pool “justifies” a premium price at 93 euros per night. The second hotel, although having 3-star facilities, lacks a swimming pool and therefore can only afford to charge 79 euros per room. Finally the hostel, lacking both facilities and a swimming pool, has to settle for the lowest room rate at 51 euros. The additive utility formulation within the conjoint model guarantees that all these elements are weighted appropriately in order to arrive at the “right” equilibrium prices.

This illustration raises a very interesting question, that is: whether the equilibrium prices determined via such a “theoretical” approach actually show any resemblance to the actual prices observed in real-world markets. If this is so, then this would imply a huge step forward in the validation of market equilibrium as a theoretical concept. The authors leave this empirical validation for another paper.

Discussion and Implications

This paper aimed at describing a game-theoretical framework for the study of optimal price setting in a competitive context by hotel revenue managers. The approach taken was to formulate the problem as a mathematical game where the players optimize their individual revenue functions through their price setting. The individual revenue functions for all the players are linked through their market share functions in which every player’s market share is a function of all the player’s prices and additional hotel attributes. Choice-based conjoint analysis was suggested as a means of measuring the relative weights (“part-worth’s”) that should be assigned to all the attributes and to be able to handle the inclusion of a none-option. In this way, the approach could be applied relatively straightforward in practice. For example, through a conjoint experiment it might be possible to establish a number of homogenous market segments, each of which might yield its own utility parameters that can be put into the model in order to generate multiple segment equilibria. When such segment equilibria are considered as a whole, a revenue manager might know his range of feasible and sustainable prices in relation to the prices that competitors have set.

A very important aspect of the model is that the use of a conjoint model for the market share function acknowledges the fact that customer value is typically determined by more attributes than price alone. This is in contrast to neo-classical approaches, which are of rather limited value for setting prices in daily practice, where price is the only distinguishing characteristic between product offers. Instead, using an additive utility function that captures all the important attributes (product quality, service levels etc as well as price) provides a more realistic description of the competitive landscape. It also allows for the possibility that a market can be in an equilibrium state while prices of the players actually differ from one another, which might be expected in hospitality markets where hotels with varying levels of quality and facilities compete for the same customers.

The most important contribution to the work by Choi and DeSarbo (1993) is the use of Choice-Based Conjoint Analysis compared to their traditional conjoint approach. Firstly, Choice-Based Conjoint is the modern standard in conjoint analysis and thus this paper can be seen as an update to their work. Secondly, Choice-Based Conjoint (in contrast to traditional conjoint) allows for the inclusion of a so-called “none-option” that can be used to model the tendency of people to defer or postpone their purchase if none of the available alternatives surpasses a certain utility threshold. This is not only important from a behavioural perspective (if consumers really defer or postpone choices with unacceptable alternatives then the predicted prices and revenues using a traditional conjoint approach would be upwardly biased) but also from an empirical perspective in the sense that certain markets, and especially within hospitality, are characterized by huge numbers of players which cannot be

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7 Although this might pose significant challenges from a model estimation point of view.
modeled separately. In this way, the paper greatly enhances the possibilities of applying equilibrium theory to day-to-day pricing problems in practice. It allows the aggregate effect of omitted players to be captured using the none-option which makes the practical validation of a game theoretical model much more straightforward.

An important reservation remains the validity of “market equilibrium” as a theoretical concept as there is no academic consensus on whether and when it actually exists. With the current model the task of empirical validation may have become one step nearer.

References